1. You are going to play 2 games of chess with an opponent whom you have never played against before (for the sake of this problem). Your opponent is equally likely to be a beginner, intermediate, or a master. Depending on   
   (a) What is your probability of winning the first game?   
   (b) Congratulations: you won the first game! Given this information, what is the probability that you will also win the second game   
   (c) Explain the distinction between assuming that the outcomes of the games are independent and assuming that they are conditionally independent given the opponent’s skill level. Which of these assumptions seems more reasonable, and why?

Answer:

(a) Let's assume that you have a probability of winning against a beginner, intermediate, and master player as p\_b, p\_i, and p\_m respectively. Since your opponent is equally likely to be any of the three skill levels, the probability of winning the first game can be calculated as:

P(win first game) = (1/3)\*p\_b + (1/3)\*p\_i + (1/3)\*p\_m

Note that we assume that the probability of playing against a beginner, intermediate, or master player is equally likely, which may not be realistic in real-life scenarios.

(b) Given that you won the first game, the probability of winning the second game depends on your opponent's skill level. Let's assume that the probability of winning the second game against a beginner, intermediate, and master player is p'\_b, p'\_i, and p'\_m, respectively. Then, we can use Bayes' theorem to calculate the conditional probability of winning the second game given that you won the first game:

P(win second game | win first game) = P(win first game and win second game) / P(win first game)

The numerator represents the joint probability of winning both games, which can be calculated as:

P(win first game and win second game) = (1/3)p\_bp'\_b + (1/3)p\_ip'\_i + (1/3)p\_mp'\_m

Note that the probability of playing against each skill level in the second game is the same as in the first game, as we do not have any additional information about our opponent's skill level.

Then, we can substitute the numerator and denominator in the Bayes' theorem equation and simplify:

P(win second game | win first game) = [(1/3)p\_bp'\_b + (1/3)p\_ip'\_i + (1/3)p\_mp'\_m] / [(1/3)\*p\_b + (1/3)\*p\_i + (1/3)\*p\_m]

(c) Assuming that the outcomes of the games are independent means that the outcome of one game does not affect the outcome of the other game. In other words, winning or losing the first game has no bearing on the probability of winning or losing the second game.

On the other hand, assuming that the outcomes of the games are conditionally independent given the opponent's skill level means that the outcome of one game may affect the outcome of the other game if we have additional information about our opponent's skill level. For example, if we win the first game against a beginner player, the probability of playing against a beginner player in the second game increases, which may affect the probability of winning the second game.

In this scenario, assuming that the outcomes of the games are conditionally independent given the opponent's skill level seems more reasonable, as winning or losing the first game may provide information about the opponent's skill level and affect the probability of winning the second game. However, it is important to note that this assumption may not hold in all scenarios and depends on the specifics of the problem at hand.